COMBINED MATHEMATICS - 1

අනාවරණ පරීක්ෂණය - 2023

13 ශ්‍රේණිය **පිළිතුරු පත්‍රය**

1. $f(n) = 3^{2n+2} - 8n - 9$

$f(n) = 3^4 - 8 - 9 = 81 - 8 - 9 = 64 \times 1$

$n = 1$ විට $f(n), 64$ න් බෙදේ. 5

$n = p$ විට $f(n), 64$ න් බෙදේ යැයි ගනිමු. $p \in \mathbb{Z}^+$

$f(p) = 3^{2p+2} - 8p - 9 = 64k, \ k \in \mathbb{Z}^+$ 5

$n = p + 1$ විට

$f(p+1) = 3^{2(p+1)+2} - 8(p+1) - 9$ 5

$= 3^{2p+2} \times 9 - 8(p+1) - 9$

$= 9(3^{2p+2} - 8p - 9) + 64p + 64$

$= 9 \times 64k + 64p + 64$

$= 64(9k + p + 1)$ 5

$n = p + 1$ විට ද ප්‍රකාශනය 64න් බෙදේ $n = 1$ විට ප්‍රකාශනය 64න් බෙදෙන බැවින් ග.ඇ.ම. අනුව සියලු $n \in \mathbb{Z}^+$ සඳහා ප්‍රකාශනය 64න් බෙදේ. .... 5 $\boxed{25}$

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2. $|x+2| \begin{cases} -(x+2) & x < -2 \\ (x+2) & x \geq -2 \end{cases}$

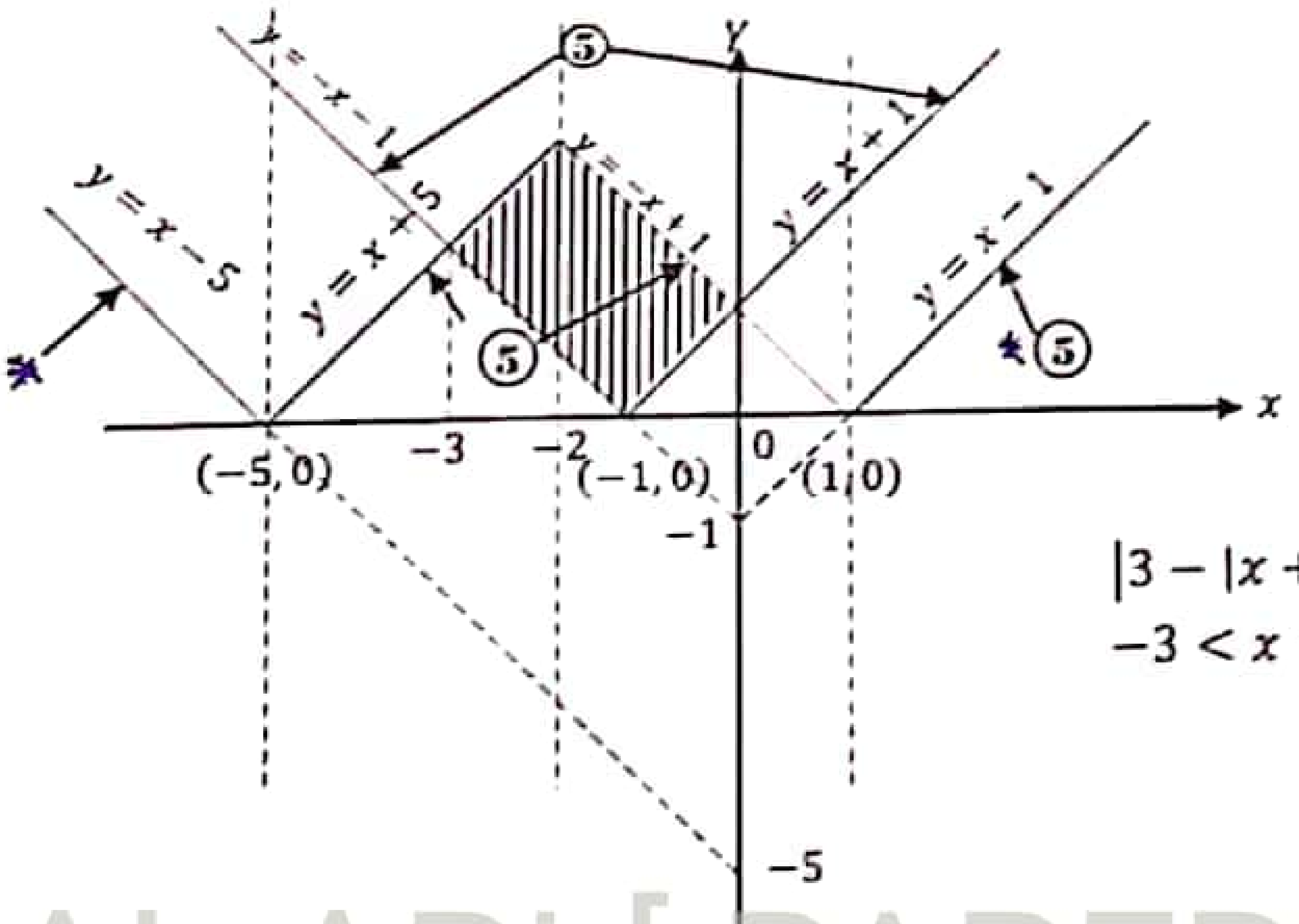
$x < -2$	$x \geq -2$
$y = \lvert 3 + x + 2 \rvert$	$y = \lvert 3 - (x+2) \rvert$
$y = \lvert x + 5 \rvert$	$y = \lvert 1 - x \rvert$

$|x+5| \begin{cases} -(x+5) & x < -5 & y = -x - 5 \\ (x+5) & x \geq -5 & y = x + 5 \end{cases}$ $(-5, 0)$

$|1-x| \begin{cases} +(1-x) & x < 1 & y = 1 - x \\ -(1-x) & x \geq 1 & y = x - 1 \end{cases}$ $(1, 0)$

$y = |x+1|$

$|x+1| \begin{cases} -(x+1) & x < -1 & y = -x - 1 \\ (x+1) & x \geq -1 & y = x + 1 \end{cases}$ $(-1, 0)$



$(-5,0)$
$(-1,0)$
$(-2,3)$
$(1,0)$ 5

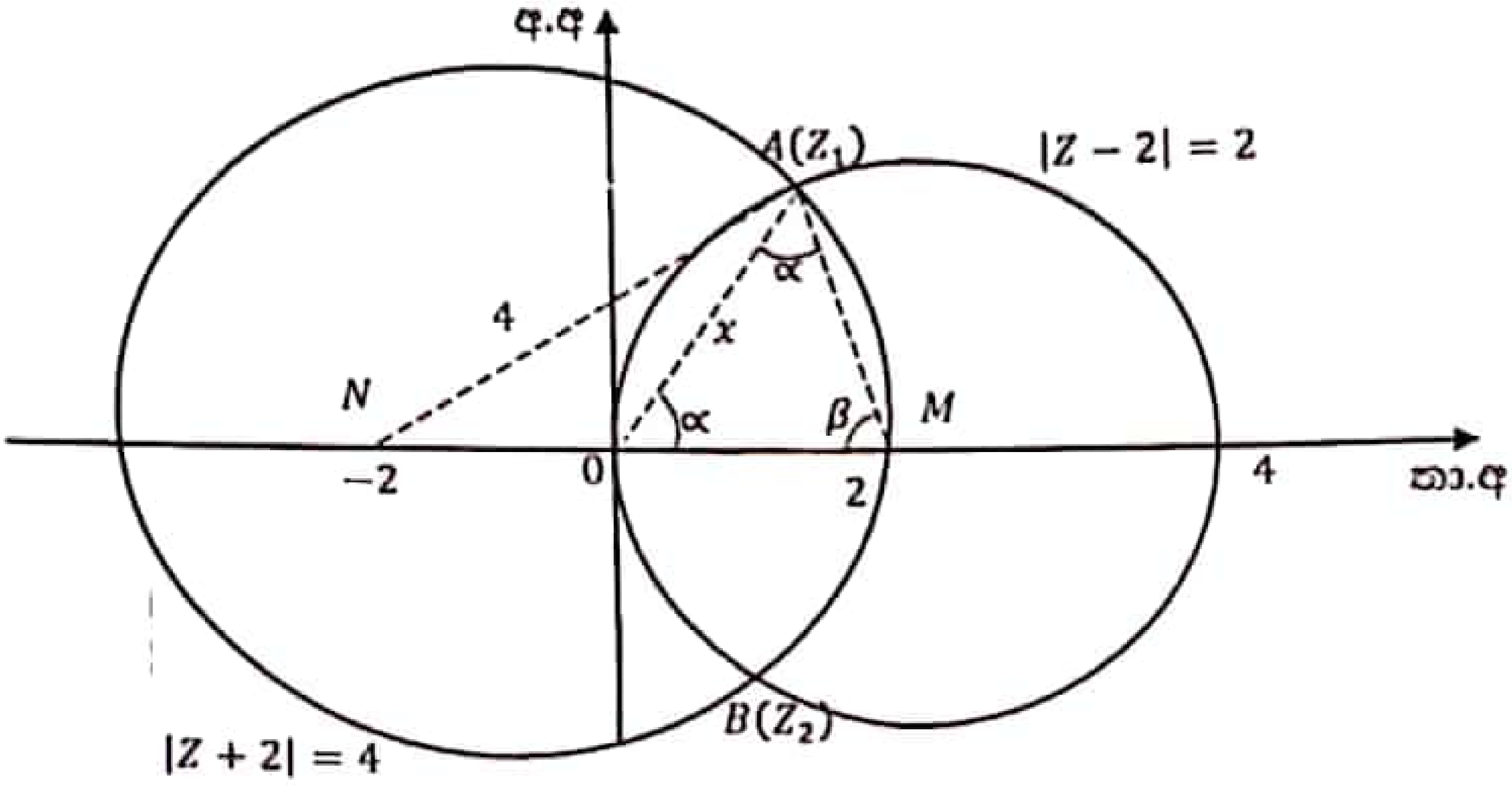
$|3 - |x+2|| > |x+1|$

$-3 < x < 0, \ x \in \mathbb{R}$ 5

$\boxed{25}$

3. $|z-2| = 2$ කේන්ද්‍රය $(2,0)$ අරය 2

$|z+2| = 4$ කේන්ද්‍රය $(-2,0)$ අරය 4



කේන්ද්‍ර අරයන් වෘත්ත දෙකට } 5

වෘත දෙකේ කේන්ද්‍රය හා අරය සමඟ රූපයට 5

$AM = 8\cos\beta = 2$ $\quad\quad$ $\pi - \beta = 2\alpha$

$\cos\beta = \frac{1}{4}$, $\quad\quad$ $-\cos\beta = 2\cos^2\alpha - 1$

$OM = x\cos\alpha + 2\cos\beta$ $\quad\quad$ $\cos^2\alpha = \frac{3}{8}$

$2 = x\sqrt{\frac{3}{8}} + \frac{2}{4} \Rightarrow x = \sqrt{6}$ $\quad\quad$ $\cos\alpha = \sqrt{\frac{3}{8}}$ 5

$x\cos\alpha = \frac{3}{2}$, $\ x\sin\alpha = \sqrt{6}\frac{\sqrt{5}}{\sqrt{8}} = \sqrt{\frac{30}{8}}$ ----- 5

A ලක්ෂයෙන් නිරූපිත සංකීර්ණ සංඛ්‍යාව $z_1 = \frac{3}{2} + \frac{\sqrt{15}}{2}i$

B ලක්ෂයෙන් නිරූපිත සංකීර්ණ සංඛ්‍යාව $z_2 = \frac{3}{2} - \frac{\sqrt{15}}{2}i$ } 5

$|\bar{Z} - 2| = |\overline{Z-2}| = |Z-2| = 2$

$|\bar{Z} + 2| = |\overline{Z+2}| = |Z+2| = 4$ බැවින් $|\bar{Z}-2| = 2$ හා $|\bar{Z}+2| = 4$ පථවල ඡේදන ලක්ෂ්‍යය මගින් නිරූපිත සංකීර්ණ සංඛ්‍යාද ඉහත Z_1 හා Z_2 ම වේ. 5 $\boxed{25}$

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4. $\left(6-\frac{5}{x}\right)(1+x)^n = \left(6-\frac{5}{x}\right)(1+{}^nC_1x+{}^nC_2x^2+ \ldots\ldots + {}^nC_{n-2}x^{n-2}+{}^nC_{n-1}x^{n-1}+{}^nC_nx^n)$

x^{n-2} හි සංගුණකය $\quad\quad$ ප්‍රසාරණයට 5

$6\,{}^nC_{n-2} - 5\,{}^nC_{n-1} = 35$ 10

$6\frac{n!}{(n-2)!2!} - 5\frac{n!}{(n-1)!1!} = 35$

$3n(n-1) - 5n = 35$

$3n^2 - 8n - 35 = 0$

$(3n+7)\ (n-5) = 0$ 5

$n = -\frac{7}{3}$ හෝ $n = 5$

$n \in \mathbb{Z}^+$ බැවින් $n = 5$ .... 5 $\boxed{25}$

5. $\lim_{x \to 0} \frac{\sin 3x}{(1-\cos 3x)} \left\{(27+x)^{\frac{1}{3}} - 3\right\}$

$= \lim_{x \to 0} \frac{2\sin\frac{3x}{2}\cos\frac{3x}{2}}{2\sin^2\frac{3x}{2}}$ (5) $\left\{\frac{(27+x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{x}\right\} \times x$ (5)

$= \lim_{x \to 0} \frac{\cos\frac{3x}{2}}{\frac{\sin\frac{3x}{2}}{x}} \left\{\frac{(27+x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{27+x-27}\right\}$ (5)

$= \frac{\lim_{x\to 0} \cos\frac{3x}{2}}{\frac{3}{2} \times \lim_{x\to 0} \frac{\sin\frac{3x}{2}}{\frac{3}{2}\times x}} \times \lim_{x \to 0} \frac{(27+x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{(27+x)-27}$ (5)

$x \to 0$ විට $\frac{3}{2}x \to 0$ හා $(27+x) \longrightarrow 27$ බැවින්

$= \frac{2}{3} \frac{\cos 0}{1} \times \frac{1}{3}\, 27^{-\frac{2}{3}}$ (5)

$= \frac{2}{3} \times 1 \times \frac{1}{3} \times \frac{1}{9}$

$= \frac{2}{81}$ $\boxed{25}$

6. $x = 2t^2 \qquad y = 4t$

$\frac{dx}{dt} = 4t \qquad \frac{dy}{dt} = 4$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 4 \times \frac{1}{4t} = \frac{1}{t}$ .... (5)

අභිලම්බයේ අනුක්‍රමණය m වේ.

$m \times \frac{1}{t} = -1$

$m = -t$

∴ අභිලම්බයේ සමීකරණය

$(y - 4t) = -t(x - 2t^2)$ .... (5)

$tx + y - 4t - 2t^3 = 0$

$t = 1$ විට අභිලම්බයේ සමීකරණය

$x + y - 6 = 0$ .... (5)

මෙම රේඛාව නැවත T පරාමිතික ලක්ෂයේදී වක්‍රය හමුවේ යැයි ගනිමු.

$x = 2T^2 \quad y = 4T$

$2T^2 + 4T - 6 = 0$

$T^2 + 2T - 3 = 0$

$(T+3)(T-1) = 0$ .... (5)

$T = -3$ හෝ $T = 1$

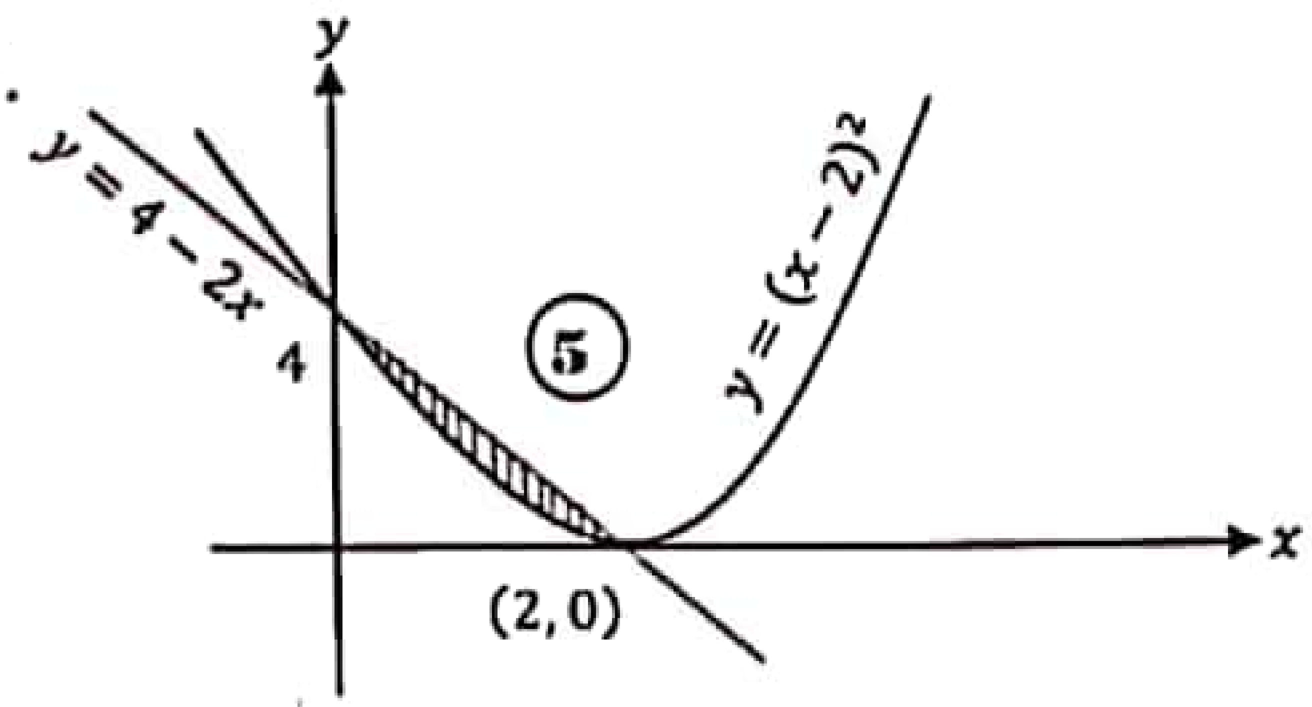
$T = 1$ යනු දී ඇති පරාමිතික අගයයි.

∴ $T = -3$

∴ $T = -3$ ලක්ෂයේදී අභිලම්බය නැවත වක්‍රය හමුවේ. .... (5) $\boxed{25}$

CamScanner

7.



පරිමාව $= \int_0^2 \pi (4-2x)^2 dx - \int_0^2 \pi (x-2)^4 dx$

5 5

$= \pi \int_0^2 (16 - 16x + 4x^2) dx - \pi \int_0^2 (x-2)^4 dx$

$= \pi \left\{ 16[x]_0^2 - 16\left[\frac{x^2}{2}\right]_0^2 + 4\left[\frac{x^3}{3}\right]_0^2 - \pi \frac{[(x-2)^5]_0^2}{5 \times 1} \right\}$ 5

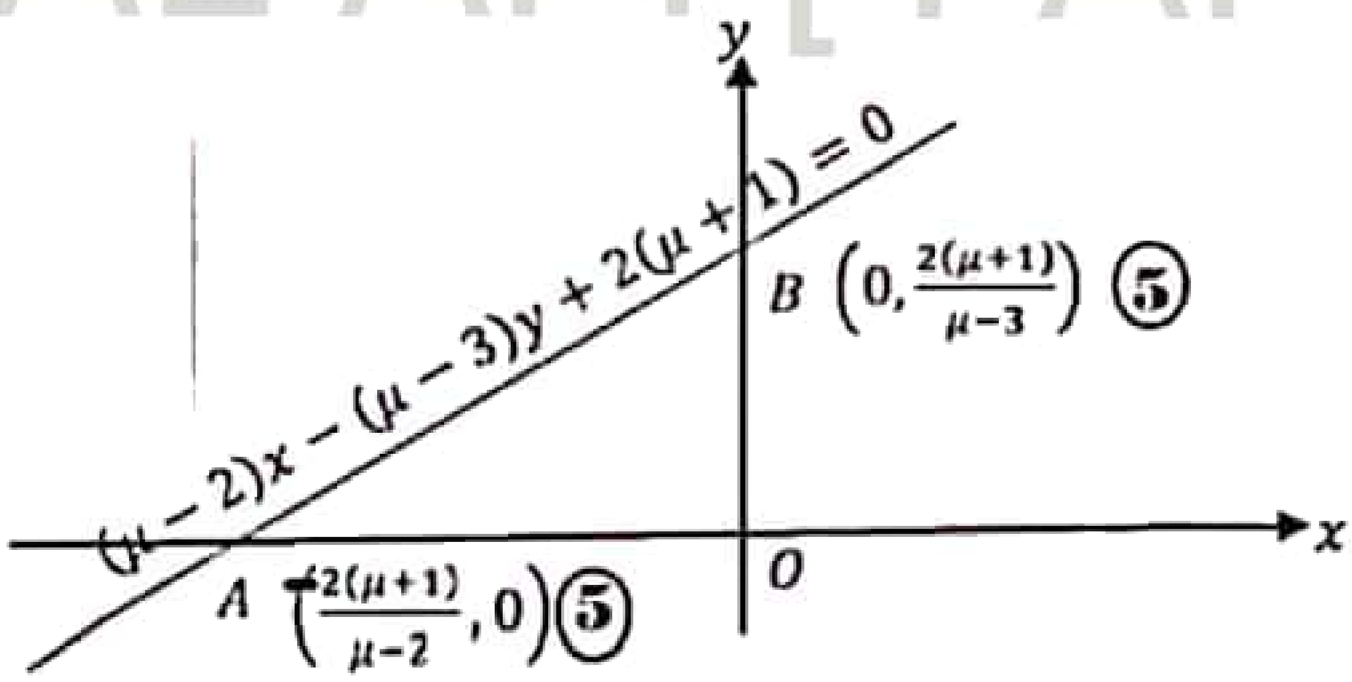
$= \pi \left\{ 16(2-0) - 8(4-0) + \frac{4}{3}(8-0) \right\} - \frac{\pi}{5}\{0 + 32\}$

$= \pi \left\{ 32 - 32 + \frac{32}{3} - \frac{32}{5} \right\}$

$= 32\pi \left(\frac{1}{3} - \frac{1}{5} \right)$

$= \frac{64}{15}\pi$ ඝන ඒකක .... 5 $\boxed{25}$

8.



$AOB \ \Delta \qquad = 2$

$\frac{1}{2} \times OA \times OB \qquad = 2$ 5

$\frac{1}{2} \times \frac{2(\mu+1)}{\mu-2} \times \frac{2(\mu+1)}{\mu-3} \qquad = 2$ 5

$(\mu+1)^2 = (\mu-2)(\mu-3)$

$\mu^2 + 2\mu + 1 \qquad = \mu^2 - 5\mu + 6$

$7\mu \qquad = 5$

$\mu \qquad = \frac{5}{7}$ .... 5 $\boxed{25}$

9. $S_1 = x^2 + y^2 + 2x - 2y - 3 = 0$

$O_1(-1,\ 1)$ $r_1 = \sqrt{1^2 + (-1)^2 - (-3)} = \sqrt{5}$ කේන්ද්‍රයට හා අරයට (5)

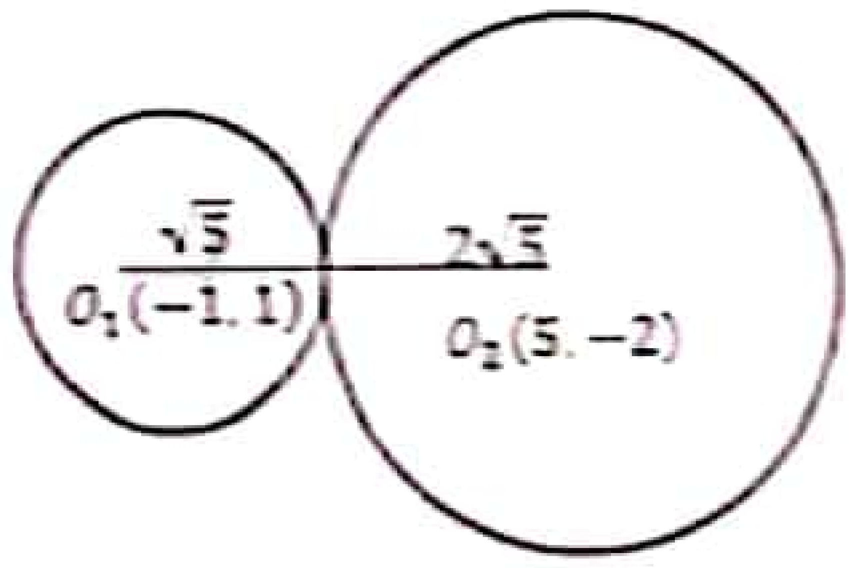
$S_2 = x^2 + y^2 - 10x + 4y + 9 = 0$

$O_2(5, -2)$ $r_2 = \sqrt{(-5)^2 + 2^2 - 9} = 2\sqrt{5}$ කේන්ද්‍රයට හා අරයට (5)

$O_1 O_2 \quad = \sqrt{(5+1)^2 + (-2-1)^2}$

$\quad = \sqrt{3^2(2^2 + 1^2)} = 3\sqrt{5}$ ______ (5)

$O_1 O_2 \quad = r_1 + r_2$ බැවින් වෘත්ත දෙක බාහිරව ස්පර්ශ වේ.



$O_1A : AO_2 \quad = \sqrt{5} : 2\sqrt{5}$

$\quad = 1 : 2$ ______ (5)

$A = \left(\frac{2(-1)+1\times 5}{1+2}, \frac{2\times 1+1(-2)}{1+2}\right)$

$= (1, 0)$ ______ (5) $\boxed{25}$

10. $\sin^4 x + \cos^4 x + \sin 2x + b = 0$

$(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x + \sin 2x + b = 0$ .... (5)

$1 - \frac{1}{2}\sin^2 2x + \sin 2x + b = 0$

$\sin^2 2x - 2\sin 2x - 2b - 2 = 0$

$\sin 2x \quad = \frac{2 \pm \sqrt{4 - 4(-2b-2)}}{2\times 1}$

$\sin 2x \quad = 1 \pm \sqrt{3 + 2b}$

$\sin 2x \leq 1$ බැවින් $\sin 2x = 1 - \sqrt{3 + 2b}$ ______ (5)

$\therefore \sin 2x = 1 - \sqrt{3 + 2b}$ .... (5)

$|1 - \sqrt{3 + 2b}| \leq 1$ හා $|\sqrt{3 + 2b} - 1| \leq 1$

$1 - \sqrt{3 + 2b} \leq 1$ හා $\sqrt{3 + 2b} - 1 \leq 1$

$3 + 2b \geq 0$ (5) $\qquad 3 + 2b \leq 4$ (5)

$b \geq -\frac{3}{2} \qquad b \leq \frac{1}{2}$ $\boxed{25}$

$\therefore -\frac{3}{2} \leq b \leq \frac{1}{2}$

11. a) $f(x) = ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a} \qquad \alpha + \beta = \frac{c}{a}$ ....දෙකටම.............................. 5

$(\alpha - 1)(\beta - 1) \quad = \alpha\beta - (\alpha + \beta) + 1$ 5

$= \frac{c}{a} + \frac{b}{a} + 1$

$= \frac{c+b+a}{a}$ .... 5 $\boxed{15}$

$\frac{1}{\alpha-1} + \frac{1}{\beta-1} \quad = \frac{\beta-1+\alpha-1}{(\alpha-1)(\beta-1)}$ 5

$= \frac{\frac{-b}{a}-2}{\frac{a+b+c}{a}}$

$= \frac{-b-2a}{a+b+c} = \frac{-(b+2a)}{a+b+c}$ 5 $\boxed{10}$

$\frac{1}{\alpha-1}$ හා $\frac{1}{\beta-1}$ මූල වන වර්ගජ සමීකරණය

$$\left(x - \frac{1}{\alpha-1}\right)\left(x - \frac{1}{\beta-1}\right) = 0$$

$$x^2 - \left(\frac{1}{\alpha-1} + \frac{1}{\beta-1}\right)x + \frac{1}{(\alpha-1)(\beta-1)} = 0$$

10 දෙකෙන් එකකට

$$x^2 + \left(\frac{b+2a}{a+b+c}\right)x + \frac{a}{a+b+c} = 0$$

$(a+b+c)x^2 + (b+2a)x + a = 0$ .... 10 $\boxed{20}$

$f(x) = ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$

$= a\left\{x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right\}$

$= a\left\{\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2-4ac)}{4a^2}\right\}$ 10

$x = \frac{-b}{2a}$ විට $f(x)$ අවම වේ.

$f(x)$ අවම $= \frac{-(b^2-4ac)}{4a}$

$\lambda = -\frac{(b^2-4ac)}{4a}$ 10 $\boxed{20}$

$\mu = \frac{-\{(2a+b)^2 - 4(a+b+c)a\}}{4(a+b+c)}$ 10

$\mu = \frac{-\{4a^2+4ab+b^2-4a^2-4ab-4ac\}}{4(a+b+c)}$

$\mu = \frac{-\{b^2-4ac\}}{4(a+b+c)}$ 5

$\frac{\lambda}{\mu} = \frac{-\frac{(b^2-4ac)}{4a}}{\frac{-\{b^2-4ac\}}{4(a+b+c)}} = \frac{a+b+c}{a}$ .... 5 $\boxed{20}$

b) $f(x) = x^4 + ax^3 + bx^2 - 10x + 4$

$(x-1)$ සාධකයක් බැවින් $f(1) = 0$ වේ.

$\left.\begin{array}{l} f(1) = 1 + a + b - 10 + 4 \\ 0 = a + b - 5 \end{array}\right\}$ 5

$a + b = 5 \longrightarrow (1)$ 5

$(x+1)$ න් බෙදූ විට ශේෂය $f(-1) = 28$ 5

$f(-1) = 1 - a + b + 10 + 4$

$28 = -a + b + 15$

$-a + b = 13 \longrightarrow (2)$ 5

$(1) + (2)$ න්

$2b = 18$ $\quad (1)$ න් $\quad a + 9 = 5$

$b = 9$ 5 $\qquad a = -4$ 5

$\boxed{30}$

$f(x) = x^4 - 4x^3 + 9x^2 - 10x + 4$

$(x-1)$ යනු සාධකයකි. 5

ඉතිරි සාධකය

$$\begin{array}{r|l} & x^3 - 3x^2 + 6x - 4 \\ \hline x - 1 & x^4 - 4x^3 + 9x^2 - 10x + 4 \\ & \underline{x^4 - x^3} \\ & \quad -3x^2 + 9x^2 - 10x + 4 \\ & \quad \underline{-3x^3 + 3x^2} \\ & \qquad\qquad 6x^2 - 10x + 4 \\ & \qquad\qquad \underline{6x^2 - 6x} \\ & \qquad\qquad\qquad -4x + 4 \\ & \qquad\qquad\qquad \underline{-4x + 4} \\ & \qquad\qquad\qquad\qquad 0 \end{array}$$

.... 5

5

$f(x) = (x-1)(x^3 - 3x^2 + 6x - 4)$

$g(x) = x^3 - 3x^2 + 6x - 4$ යැයි ගනිමු.

$g(1) = 1^3 - 3 + 6 - 4 = 0$

$(x-1)$ යනු $g(x)$ හි සාධකයකි. 5

ඉතිරි සාධක

$$\begin{array}{r|l} & x^2 - 2x + 4 \\ \hline x - 1 & x^3 - 3x^2 + 6x - 4 \\ & \underline{x^3 - x^2} \\ & \quad -2x^2 + 6x - 4 \\ & \quad \underline{-2x^2 + 2x} \\ & \qquad\qquad 4x - 4 \\ & \qquad\qquad \underline{4x - 4} \\ & \qquad\qquad\quad 0 \end{array}$$

5

$f(x) = (x-1)(x-1)(x^2 - 2x + 4)$

$(x-1)^2[(x-1)^2 + 3] \geq 0$ 5

$\therefore$ $f(x) \geq 0$ වේ. .... 5 $\quad \boxed{35}$

$x = 1$ විට සමානතාව පවතී

12. a) i) $\quad {}^{17}C_5 = \frac{17!}{5! \times 12!} = 6188$ 5 5 $\boxed{10}$

ii)

ගැ	පි
4	1
3	2
2	3
1	4

$$ {}^{7}C_4 \times {}^{10}C_1 = 35 \times 10 = 350 $$
$$ {}^{7}C_3 \times {}^{10}C_2 = 35 \times 45 = 1575 $$
$$ {}^{7}C_2 \times {}^{10}C_3 = 21 \times 120 = 2520 $$
$$ {}^{7}C_1 \times {}^{10}C_4 = 7 \times 210 = 1470 $$
$$ 5915 \text{ — } 5 $$

නිවැරදි 4 කට 15
3 කට 10
2 කට 5

$\boxed{20}$

iii)

A	B
4	1
3	2
2	3
1	4

$$ {}^{6}C_1 \times {}^{11}C_1 = 15 \times 11 = 165 $$
$$ {}^{6}C_3 \times {}^{11}C_2 = 20 \times 55 = 1100 $$
$$ {}^{6}C_2 \times {}^{11}C_3 = 15 \times 165 = 2475 $$
$$ {}^{6}C_1 \times {}^{11}C_4 = 6 \times 330 = 1980 $$
$$ 5720 \text{ — } 5 $$

නිවැරදි 4 කට 15
3 කට 10
2 කට 5

$\boxed{20}$

iv)

A: ගැ	A: පි	B: ගැ	B: පි
2	1	1	1
1	2	1	1
1	1	2	1
1	1	1	2

$$ {}^{3}C_2 \times {}^{3}C_1 \times {}^{4}C_1 \times {}^{7}C_1 \; 5 = 252 $$
$$ {}^{3}C_1 \times {}^{3}C_2 \times {}^{4}C_1 \times {}^{7}C_1 \; 5 = 252 $$
$$ {}^{3}C_1 \times {}^{3}C_1 \times {}^{4}C_2 \times {}^{7}C_1 \; 5 = 378 $$
$$ {}^{3}C_1 \times {}^{3}C_1 \times {}^{4}C_1 \times {}^{7}C_2 \; 5 = 756 $$
$$ = 1638 \; 5 $$

$\boxed{25}$

b) $\frac{4r^2+12r+6}{r(r+1)(r+2)(r+3)} = \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2} + \frac{D}{r+3}$ 5

$4r^2 + 12r + 6 = A(r+1)(r+2)(r+3) + Br(r+2)(r+3)$
$+cr(r+1)(r+3) + Dr(r+1)(r+2)$

දෙපස අනුරූප පදවල සංගුණක සැලකීමෙන්,

$'r^3'$; $A + B + C + D = 0$ $\qquad r^2$; $6A + 5B + 4C + 3D = 4$

$'r^1'$; $11A + 5B + 3C + 2D = 12$ $\qquad$ නියත, $6A = 6$

$A = 1$

$B = 1, \quad C = -1, \quad D = -1$

නිවැරදි 4 කට 10
නිවැරදි 3 කට 5

$\boxed{15}$

$$\frac{4r^2+12r+6}{r(r+1)(r+2)(r+3)} = \frac{1}{r} + \frac{1}{r+1} - \frac{1}{r+2} - \frac{1}{r+3}$$

$$V_r = \frac{1}{r} + \frac{1}{r+1} = \frac{2r+1}{r(r+1)} \quad 5$$

$$\therefore V_{r+2} = \frac{1}{r+2} + \frac{1}{r+3}$$

$$V_{r+2} = \frac{2r+5}{(r+2)(r+3)}$$

$$\therefore U_r = V_r - V_{r+2}$$

එවිට

$$U_r = V_r - V_{r+2}$$

$r = 1$ විට $\quad U_1 = V_1 - V_3$ නිවැරදි 3 කට 10

$r = 2$ විට $\quad U_2 = V_2 - V_4$ නිවැරදි 2 කට 5

$r = 3$ විට $\quad U_3 = V_3 - V_5$

$r = n-2$ විට $\quad U_{n-2} = V_{n-2} - V_n$ නිවැරදි 3 කට 10

$r = n-1$ විට $\quad U_{n-1} = V_{n-1} - V_{n+1}$ නිවැරදි 2 කට 5

$r = n$ විට $\quad U_n = V_n - V_{n+2}$

$$\sum_{r=1}^{n} U_n = V_1 - V_2 - V_{n+1} - V_{n+2} \quad 5$$

$$\sum_{r=1}^{n} U_n = \frac{3}{2} + \frac{5}{6} - \frac{(2n+3)}{(n+1)(n+2)} - \frac{(2n+5)}{(n+2)(n+3)}$$

$$= \frac{7}{3} - \frac{(2n+3)}{(n+1)(n+2)} - \frac{(2n+5)}{(n+2)(n+3)} \quad 10$$

$\boxed{40}$

$$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \frac{7}{3} - \lim_{n\to\infty}\left[\frac{2n+3}{(n+1)(n+2)} - \frac{2n+5}{(n+2)(n+3)}\right]$$

$$\sum_{r=1}^{\infty} U_r = \frac{7}{3} \quad 5$$

$n \to \infty$ විට අවසාන පද දෙක ශූන්‍ය කරා එළඹෙන නිසා අපරිමිත පද ගණනක ඓක්‍යය පරිමිත අගයක් කරා එළඹෙන බැවින් ශ්‍රේණිය අභිසාරී වේ. 5

$\boxed{10}$

$$\sum_{r=1}^{2n} U_r = V_1 + V_2 - V_{2n+1} - V_{2n+2}$$

$$= \frac{7}{3} - \frac{4n+3}{(2n+1)(2n+2)} - \frac{(4n+5)}{(2n+2)(2n+3)} \quad 5$$

$$\sum_{r=n+1}^{2n} U_r = \sum_{r=1}^{2n} U_r - \sum_{r=1}^{n} U_r$$

$$= \frac{7}{3} - \frac{4n+3}{(2n+1)(2n+2)} - \frac{4n+5}{(2n+2)(2n+3)} - \left[\frac{7}{3} - \frac{2n+3}{(n+1)(n+2)} - \frac{2n+5}{(n+2)(n+3)}\right]$$

$$\sum_{r=n+1}^{2n} U_r = \frac{(2n+3)}{(n+1)(n+2)} + \frac{(2n+5)}{(n+2)(n+3)} - \frac{(4n+3)}{(2n+1)(2n+2)} - \frac{(4n+5)}{(2n+2)(2n+3)} \quad 5$$

$\boxed{10}$

13. a) $|A| = \begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} = 2 - (-12) = 14 \neq 0 \quad \therefore A^{-1}$ පවතී. 5

$$A^{-1} = \frac{1}{14}\begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & -\frac{3}{14} \\ \frac{2}{7} & \frac{1}{7} \end{pmatrix} \quad 5$$

$$(\lambda A)^{-1} = \left[\lambda \begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}\right]^{-1} = \begin{pmatrix} 2\lambda & 3\lambda \\ -4\lambda & \lambda \end{pmatrix}^{-1} = \frac{1}{14\lambda}\begin{pmatrix} \lambda & -3\lambda \\ 4\lambda & 2\lambda \end{pmatrix} \quad 5$$

$$(\lambda A)^{-1} = \lambda^{-1}\begin{pmatrix} \frac{1}{14} & -\frac{3}{14} \\ \frac{2}{7} & \frac{1}{7} \end{pmatrix} = \frac{1}{\lambda}A^{-1} \quad 5 \qquad \lambda \neq 0$$

$$C = (3A)^{-1}B$$

$$C = \frac{1}{3}A^{-1}B = \frac{1}{3}\begin{pmatrix} \frac{1}{14} & -\frac{3}{14} \\ \frac{2}{7} & \frac{1}{7} \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 1 & 0 \end{pmatrix} \quad 5$$

$$C = \frac{1}{3}\begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{11}{7} & \frac{12}{7} \end{pmatrix} \quad 5$$

$$AC + BC = \begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{21} & \frac{3}{21} \\ \frac{11}{21} & \frac{12}{21} \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{21} & \frac{3}{21} \\ \frac{11}{21} & \frac{12}{21} \end{pmatrix} \quad 5$$

$$= \begin{pmatrix} \frac{35}{21} & \frac{42}{21} \\ \frac{7}{21} & 0 \end{pmatrix} + \begin{pmatrix} \frac{71}{21} & \frac{87}{21} \\ \frac{1}{21} & \frac{3}{21} \end{pmatrix} = \begin{pmatrix} \frac{106}{21} & \frac{129}{21} \\ \frac{8}{21} & \frac{3}{21} \end{pmatrix}$$

$$= \frac{1}{21}\begin{pmatrix} 106 & 129 \\ 8 & 3 \end{pmatrix} \quad 5$$

$$A + B = DC^{-1}$$

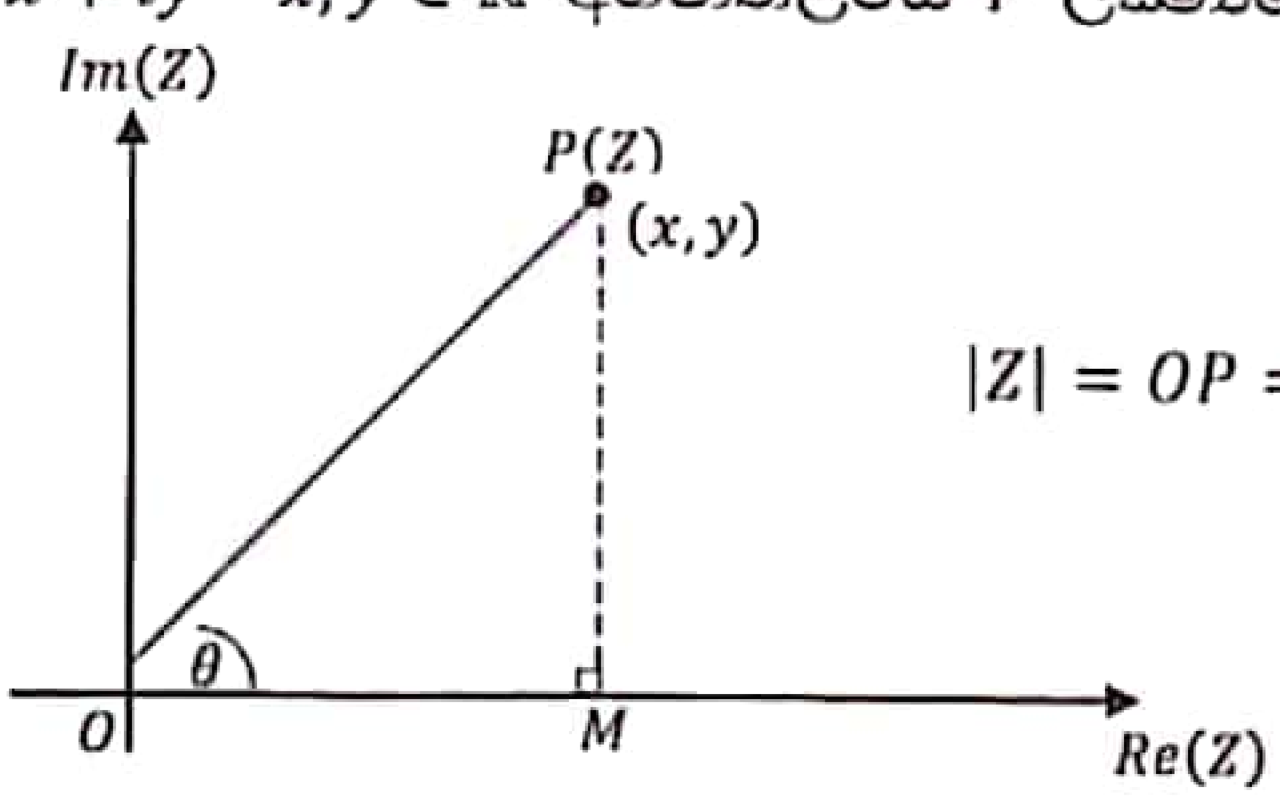
$$AC + BC = DC^{-1}C \quad 5$$

$$AC + BC = D$$

$$D = \frac{1}{21}\begin{bmatrix} 106 & 129 \\ 8 & 3 \end{bmatrix}_{2\times 2} \quad 5$$

$\boxed{50}$

b) $z = x + iy \quad x, y \in \mathbb{R}$ අගන්තලයේ P ලක්ෂයෙන් නිරූපණය කරමු.



$$|Z| = OP = \sqrt{x^2 + y^2} \quad 5$$

Z හි ප්‍රධාන විස්තාරය $Arg(Z) = M\hat{O}P = \theta \qquad -\pi \leq \theta < \pi$ 5 $\boxed{10}$

i) $|Z| = \sqrt{x^2+y^2} \geq \sqrt{x^2} = \sqrt{|x|^2} = |x|$ 5 5

$\therefore \ |x| \leq |Z|$

$|Re(Z)| \leq |Z|$

$-|Z| \leq Re(Z) \leq |Z|$ —— 5

$\boxed{15}$

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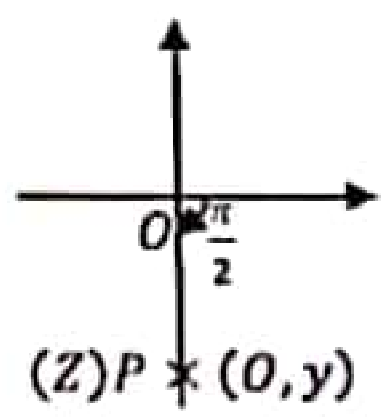
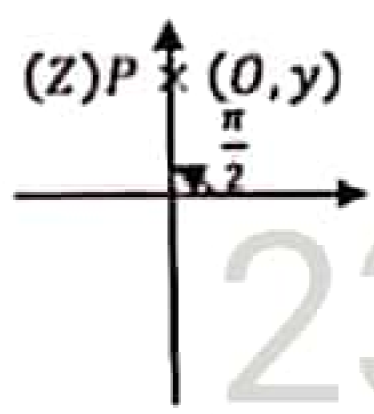
ii) Z සංකීර්ණ සංඛ්‍යාව හුදෙක් අතාත්වික නම් $x = 0$ එවිට $Z = iy, y \in \mathbb{R} - \{0\}$

$y < 0$ විට $\qquad$ $y > 0$ විට





$Arg(Z) = \frac{-\pi}{2}$ 5 $\qquad$ $Arg(Z) = \frac{\pi}{2}$ 5

Z සංකීර්ණ සංඛ්‍යාව හුදෙක් අතාත්වික නම් $Arg(Z) = \pm\frac{\pi}{2}$ 5

$|ArgZ| = \frac{\pi}{2}$ $\boxed{15}$

C) n ධන පූර්ණ සංඛ්‍යාවක් සඳහා ද' මුවාවර් ප්‍රමේයය

$(Cos\theta + i\, sin\theta)^n = Cosn\theta + isin\, n\theta$ 5

n ධන පූර්ණ සංඛ්‍යාවක් සඳහා ද්විපද ප්‍රමේයය

$(a+b)^n = \sum_{r=0}^{n} {}^{n}C_r a^{n-r} b^r$ මෙහි ${}^{n}C_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$ 5

$Z = cos\theta + i\, sin\theta$

$\frac{1}{Z} = \frac{1}{Cos\theta + i\, Sin\theta} = \frac{Cos\theta - i\, Sin\theta}{Cos^2\theta - i^2 Sin^2\theta}$

$\frac{1}{Z} = Cos\theta - i\, sin\theta$

$Z^n = Cosn\theta + i\, sinn\theta \longrightarrow (1)$ 5

$\frac{1}{Z^n} = Cosn\theta - i\, sinn\theta \longrightarrow (2)$ 5

(1) + (2) න්

$Z^n + \frac{1}{Z^n} = 2Cos(n\theta)$ 5 $\qquad Z + \frac{1}{Z} = 2Cos\theta$

(1) − (2) න්

$Z^n - \frac{1}{Z^n} = 2iSin(n\theta)$ 5 $\qquad Z - \frac{1}{Z} = 2iSin\theta$

ද්විපද ප්‍රසාරණයෙන්

$\left(Z + \frac{1}{Z}\right)^6 = {}^6C_0 Z^6 + {}^6C_1 Z^5.\frac{1}{Z} + {}^6C_2 Z^4.\frac{1}{Z^2} + {}^6C_3 Z^3.\frac{1}{Z^3} + {}^6C_4 Z^2.\frac{1}{Z^4} + {}^6C_5 Z.\frac{1}{Z^5} + {}^6C_6.\frac{1}{Z^6}$ 5

$= \left(Z^6 + \frac{1}{Z^6}\right) + 6\left(Z^4 + \frac{1}{Z^4}\right) + 15\left(Z^2 + \frac{1}{Z^2}\right) + 20$

$(2Cos\theta)^6 = 2Cos6\theta + 6 \times 2Cos4\theta + 15 \times 2Cos2\theta + 20$ 5

$32cos^6\theta = Cos6\theta + 6cos4\theta + 15cos2\theta + 10 \longrightarrow (A)$

$\left(Z - \frac{1}{Z}\right)^6 = Z^6 - 6Z^4 + 15Z^2 - 20 + \frac{15}{Z^2} - \frac{6}{Z^4} + \frac{1}{Z^6}$ 5

$$= \left(Z^6 + \frac{1}{Z^6}\right) - 6\left(Z^4 + \frac{1}{Z^4}\right) + 15\left(Z^2 + \frac{1}{Z^2}\right) - 20$$

$$(2i\sin\theta)^6 = 2\cos 6\theta - 6.\ 2\cos 4\theta + 15.\ 2\cos 2\theta - 20$$

$$-32\sin^6\theta = \cos 6\theta - 6.\ 2\cos 4\theta + 15.\ \cos 2\theta - 10 \longrightarrow B \qquad 5$$

$A + B$

i) $32(\cos 6\theta - \sin^6\theta) = 2\cos 6\theta + 30\cos 2\theta$

$16(\cos^6\theta - \sin^6\theta) = \cos 6\theta + 15\cos 2\theta$ 5

iii) $A - B$

$32(\cos^6\theta + \sin^6\theta) = 12\cos 4\theta + 20$

$8(\cos^6\theta + \sin^6\theta) = 3\cos 4\theta + 5$ 5 $\boxed{60}$

14.a) $f(x) = \frac{(2x+1)(x-2)}{(x+1)^2}$ 5

$$f^1(x) = \frac{(x+1)^2\{2x+1+(x-2)2\} - (2x+1)(x-2).2(x+1)}{(x+1)^4} \quad 5 \quad 5$$

$$= \frac{(x+1)[(x+1)(4x-3)] - 2(2x+1)(x-2)}{(x+1)^4}$$

$$= \frac{7x+1}{(x+1)^3} \quad 5$$

$x = -\frac{1}{7}$ විට $f^1(x) = 0$ වේ. $\boxed{20}$

$x = -1$ විට $f^1(x)$ අර්ථ නොදැක්වේ.

$-\infty < x < -1$	$-1 < x < -\frac{1}{7}$	$-\frac{1}{7} < x < \infty$
$f^1(x) > 0$ 5	$f^1(x) < 0$ 5	$f^1(x) > 0$ 5

$f(x)$ වැඩි වන ප්‍රාන්තරය $(-\infty, -1) \cup \left(-\frac{1}{7}, \infty\right)$ 5

$f(x)$ අඩු වන ප්‍රාන්තරය $\left(-1, -\frac{1}{7}\right)$ 5

$$x = -\frac{1}{7} \text{ විට } f(x) = \frac{\left(\frac{-2}{7}+1\right)\left(\frac{-1}{7}-2\right)}{\left(-\frac{1}{7}+1\right)^2} = \frac{5}{7} \times \frac{-15}{7} \times \frac{49}{36} = \frac{-25}{12}$$

අවම ලක්ෂ්‍යය $\left(-\frac{1}{7}, \frac{-25}{12}\right)$ 5 $\boxed{30}$

$$f(x) = \frac{2x^2-3x-2}{x^2+2x+1} = \frac{x^2\left(2-\frac{3}{x}-\frac{2}{x^2}\right)}{x^2\left(1+\frac{2}{x}+\frac{1}{x^2}\right)} \qquad f(x) = \frac{(2x+1)(x-2)}{(x+1)^2}$$

$x \to +\infty$ විට $f(x) \to 2$

$x \to -\infty$ විට $f(x) \to 2$ 5

$x \to -1 - h$ විට $f(x) + \infty$

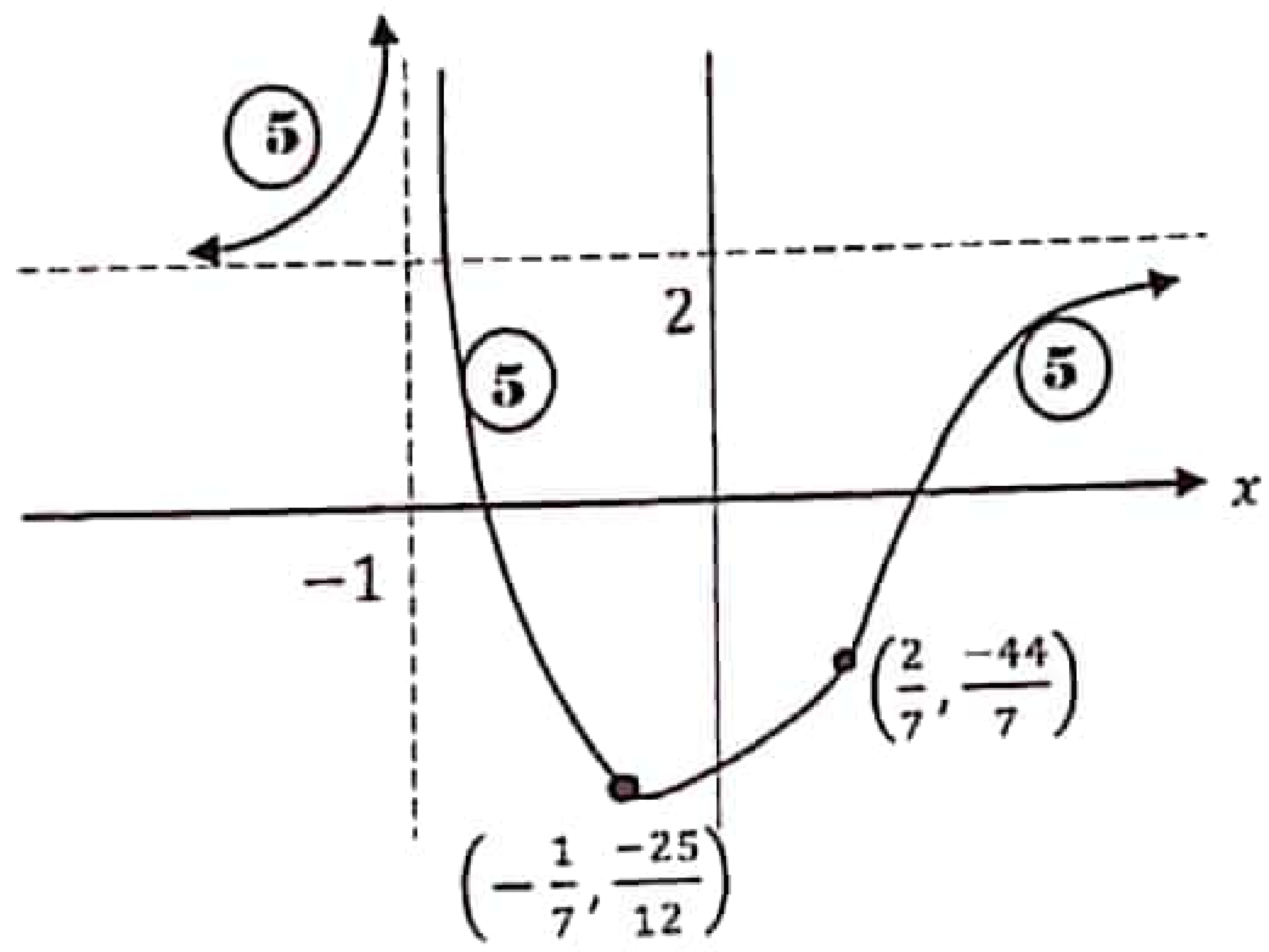
$x \to -1 + h$ විට $f(x) + \infty$ 5

$$f^{11}(x) = \frac{2(2-7x)}{(x+1)^4}$$

$x = \frac{2}{7}$ විට නතිවර්තන ලක්ෂ්‍යයක් ඇත.

$-\infty < x < -1$	$-1 < x < \frac{2}{7}$	$\frac{2}{7} < x < \infty$
$f^{11}(x) > 0$ උඩු අවතල	$f^{11}(x) > 0$ උඩු අවතල 5	$f^{11}(x) < 0$ යටි අවතල 5

නති වර්තන ලක්ෂ්‍යය $= \left(\frac{2}{7}, \frac{-44}{27}\right)$ 5



$\boxed{40}$

b) වර්ගඵලය $= 144$

$5y \times x + 5y \times x + y \times 2x = 144$ 5

$12xy = 144$

$xy = 12$ 5

$L = 2 \times 5y + 2 \times 4y + 4x \times 2$.......... 5

$= 18y + 8x$

$= 18 \times \frac{12}{x} + 8x$

$L = \frac{216}{x} + 8x$ 5

$\boxed{20}$

$\frac{dL}{dx} = 216 \times -1x^2 + 8$ 10

$= \frac{-216}{x^2} + 8 = 8\left(1 - \frac{27}{x^2}\right)$

$= 8\frac{\left(x^2 - (3\sqrt{3})^2\right)}{x^2}$

$= \frac{8(x - 3\sqrt{3})(x + 3\sqrt{3})}{x^2}$ 5

$\frac{dL}{dx} = 0$ වන්නේ $x = 3\sqrt{3}$ හා $x = -3\sqrt{3}$ විටදී

$x > 0$ බැවින් $x = 3\sqrt{3}$ 5

$0 < x < 3\sqrt{3}$	$3\sqrt{3} < x$
$\frac{dL}{dx} < 0$ 5	$\frac{dL}{dx} > 0$ 5

$x = 3\sqrt{3}m$ විට L අවම වේ.

$\therefore$ L අවම වන විට

$x = 3\sqrt{3}m$ 5

$y = \frac{12}{3\sqrt{3}} = \frac{4}{\sqrt{3}}m$ 5

$\boxed{40}$

15. (a) $4x^2+5x-2 = A(x+1)(x^2-x+1) + B(x^2-x+1) + (2x+C)(x+1)^2$

$x=-1$ විට

$-3 = 3B$

$B=-1$

x^3 සංගුණක $0 = A+2 \qquad A=-2$

x^0 සංගුණක $-2 = A+B+C \qquad C=1$

නිවැරදි
3 කට (10)
2 කට (5)

$$\frac{4x^2+5x-2}{(x+1)(x^3+1)} = \frac{-2(x+1)(x^2-x+1)-1(x^2-x+1)+(2x+1)(x+1)^2}{(x+1)^2(x^2-x+1)}$$

$$= \frac{-2}{(x+1)} - \frac{1}{(x+1)^2} + \frac{2x+1}{(x^2-x+1)} \quad (5)$$

15

$$\int \frac{4x^2+5x-2}{(x+1)(x^3+1)}dx = \int \frac{-2}{x+1} - \frac{1}{(x+1)^2} + \frac{2x+1}{x^2-x+1}dx$$

$$= -2\int \frac{1}{x+1}dx - \int (x+1)^{-2}dx + \int \frac{2x-1+2}{x^2-x+1}dx \quad (5)$$

$$= -2\ln|x+1| - \frac{(x+1)^{-1}}{-1} + 1\int \frac{2x-1}{x^2-x+1}dx + 2\int \frac{1}{(x^2-x+1)}dx$$

$$= -2\ln|x+1| + \frac{1}{(x+1)} + \ln|x^2-x+1| 2\int \frac{1}{(x-\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

නිවැරදි
5 කට (20)
4 කට (15)
3 කට (10)
2 කට (5)

$$-2\ln|x+1| + \frac{1}{x+1} + \ln|x^2-x+1| + 2\,\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + \text{නියත}$$

$$-2\ln|x+1| + \frac{1}{x+1} + \ln|x^2-x+1| + \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + \text{නියත}$$

25

(b) $\dfrac{x+\sin 2x}{1+\cos 2x} = \dfrac{x+2\sin x\cos x}{2\cos^2 x}$ - (5) - (5)

$$= \frac{x}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \frac{x}{2}\sec^2 x + \tan x$$

10

$$\int_0^{\frac{\pi}{4}} \frac{x+\sin 2x}{1+\cos 2x}dx = \int_0^{\frac{\pi}{4}} \left(\frac{x}{2}\sec^2 x + \tan x\right)dx \quad - (5)$$

(5)

$$= \int_0^{\frac{\pi}{4}} \frac{x}{2}\frac{d}{dx}\tan x\,dx + \int_0^{\frac{\pi}{4}} \tan x\,dx$$

(5) (5)

$$= \left[\frac{x}{2}\tan x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x \times \frac{1}{2}\,dx + \int_0^{\frac{\pi}{4}} \tan x\,dx$$

$$= \left(\frac{\pi}{8}\times 1 - 0\right) + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}dx$$

$$\int_0^{\frac{\pi}{4}} \frac{x+\sin 2x}{1+\cos 2x}\,dx = \frac{\pi}{8} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx \quad (5)$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[\ln|\cos x|\right]_0^{\frac{\pi}{4}} \quad (5)$$

$$= \frac{\pi}{8} - \frac{1}{2}\left(\ln\frac{1}{\sqrt{2}} - \ln 1\right)$$

$$= \frac{\pi}{8} - \frac{1}{2}\left(\ln 1 - \ln\sqrt{2} - \ln 1\right)$$

$$= \frac{\pi}{8} + \frac{1}{2}\ln\sqrt{2}$$

$$A = \frac{\pi}{8} \qquad B = \frac{1}{2} \qquad C = \sqrt{2}$$

නිවැරදි

3 ක (10)

2 ක (5)

$\boxed{40}$

(c) $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$

$$\int_0^a f(a-x)dx = \int_0^a f(a-x)\frac{dx}{dy}\,dy$$

$y = a - x$ යයි ගනිමු.

$$= \int_a^0 f(y)(-1)dy \quad (5)$$

$$\left.\begin{aligned} x = 0 \text{ විට } y = a \\ x = a \text{ විට } y = 0 \end{aligned}\right\} (5)$$

$$= \int_0^a f(y)dy$$

$$= \int_0^a f(x)dx \quad (5)$$

$\frac{dy}{dx}\ -1$

$\boxed{15}$

$$I = \int_0^{\frac{\pi}{6}} \frac{x\sin 3x\cos 3x}{\sin^4 3x + \cos^4 3x}\,dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{\left(\frac{\pi}{6}-x\right)\sin\left(\frac{\pi}{2}-3x\right)\cos\left(\frac{\pi}{2}-3x\right)}{\sin^4\left(\frac{\pi}{2}-3x\right)+\cos^4\left(\frac{\pi}{2}-3x\right)}\,dx \quad (10)$$

$$= \int_0^{\frac{\pi}{6}} \frac{\left(\frac{\pi}{6}-x\right)\cos 3x\sin 3x}{\cos^4 3x+\sin^4 3x}\,dx$$

$$I = \frac{\pi}{6}\int_0^{\frac{\pi}{6}} \frac{\cos 3x\cos 3x}{\sin^4 3x+\cos^4 3x}\,dx - \int_0^{\frac{\pi}{6}} \frac{x\cos 3x\sin 3x}{\cos^4 3x+\sin^4 3x}\,dx \quad (5)$$

$$2I = \frac{\pi}{6}\int_0^{\frac{\pi}{6}} \frac{\sin 3x\cos 3x}{\sin^4 3x+\cos^4 3x}\,dx$$

$$I = \frac{\pi}{12}\int_0^{\frac{\pi}{6}} \frac{\sin 3x\cos 3x}{\sin^4 3x+\cos^4 3x}\,dx \quad (5)$$

$\boxed{20}$

$$I = \frac{\pi}{12}\int_0^{\frac{\pi}{6}} \frac{\frac{1}{2}\sin 6x}{(\sin^2 3x + \cos^2 3x)^2 - 2\sin^2 3x . \cos^2 3x}\, dx \quad (5)\ (5) \qquad = \frac{\pi}{24}\int_0^{\frac{\pi}{6}} \frac{\sin 6x}{1 - \frac{1}{2}\sin^2 6x}\, dx$$

$$I = \frac{\pi}{24}\int_0^{\frac{\pi}{6}} \frac{\sin 6x}{1 - \frac{1}{2}(1 - \cos^2 6x)}\, dx$$

$$= \frac{\pi}{24}\int_0^{\frac{\pi}{6}} \frac{2\sin 6x}{1 + \cos^2 6x}\, dx \quad (5)$$

$$I = \frac{\pi}{12}\int_0^{\frac{\pi}{6}} \frac{\sin 6x}{1 + \cos^2 6x}\, dx$$

$$I = \frac{\pi}{12}\int_1^{-1} \frac{\sin 6x}{1 + \cos^2 6x}\ \frac{dx}{dt}\ dt$$

$$I = \frac{\pi}{12}\int_1^{-1} \frac{\sin 6x}{1 + \cos^2 6x}\ \frac{1}{-6\sin 6x}\ dt$$

$$I = \frac{\pi}{72}\int_{-1}^{1} \frac{1}{1 + t^2}\ dt \quad (5)$$

$$I = \frac{\pi}{72}\ [\tan^{-1} t]_{-1}^{1}$$

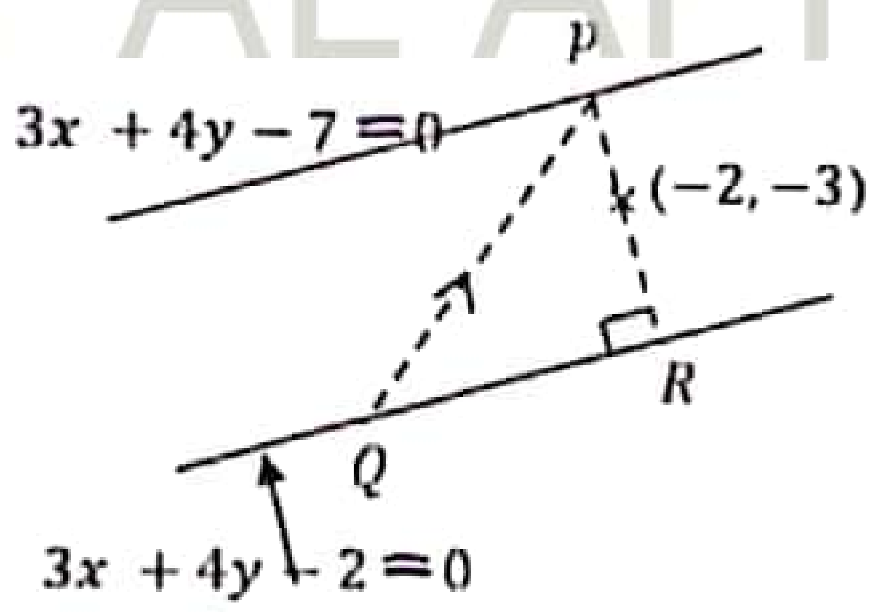
$$I = \frac{\pi}{72}\left(\tan^{-1} 1 - \tan^{-1}(-1)\right)$$

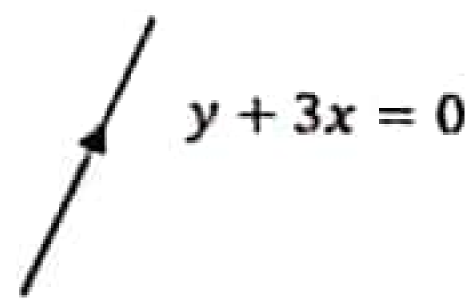
$$= \frac{\pi}{72}\left(\frac{\pi}{4} + \frac{\pi}{4}\right)$$

$$= \frac{\pi^2}{144} \quad (5)$$

$\boxed{25}$

16. (a)





P හි ඛණ්ඩාංක

$3x + 4y = 7 \quad - \ ①$

$4x - 3y = 1 \quad - \ ②$

$① \times 3 + ② \times 4$

$25x = 25$

$x = 1$

$y = 1$

$p \equiv (1,1) \quad (10)$

$\boxed{20}$

PR හි අනුක්‍රමණය

$m_1 \times m_2 = -1$

$\frac{-3}{4} \times m_2 = 1$

$m_2 = \frac{4}{3} \quad (5)$

PR හි සමීකරණය

$y + 3 = \frac{4}{3}(x + 2) \quad (5)$

$3(y + 3) = 4(x + 2)$

$4x - 3y - 1 = 0$

PQ හිසමීකරණය

$y + 3x + k = 0$

$p\,(1, 1)$ හරහා යන බැවින්

$1 + 3 + k = 0$

$k = -4$

$PQ = y + 3x - 4 = 0 \quad (5)$

Q හි ඛණ්ඩාංකය

$3x + 4y = 2 \quad \to (3)$

$3x + y = 4 \quad \to (4)$

$(3) - (4) \times 4 \quad -9x = -4$

$x = \frac{14}{9}$

$y = \frac{-2}{3}$

$Q \equiv \left(\frac{14}{9}, \frac{-2}{3}\right)$ 10 $\boxed{15}$

PQRΔ වි.ඵ $= 1$

$\frac{1}{2} \times QR \times PR = 1$

$\frac{1}{2} \times QR \times \frac{|(-7-(-2)|}{\sqrt{3^2+4^2}} = 1$ 5

$QR = 2$ 5

R හි ඛණ්ඩාංක පරාමිතිය ඇසුරින්

$$\frac{y + \frac{2}{3}}{x - \frac{14}{9}} = -\frac{3}{4}$$ 5

$\frac{y+\frac{2}{3}}{-3} = \frac{x-\frac{14}{9}}{4} = t$ යැයි ගනිමු

$y = -\frac{2}{3} - 3t \quad x = \frac{14}{9} + 4t$

$$R = \left\{\frac{14}{9} + 4t, -\frac{2}{3} - 3t\right\}$$ 5

QR = 2 බැවින්

$QR^2 = 4$

$$\left(\frac{14}{9} + 4t - \frac{14}{9}\right)^2 + \left(-\frac{2}{3} - 3t + \frac{2}{3}\right)^2 = 4$$ 10

$16t^2 + 9t^2 = 4$

$t = \pm\frac{2}{5}$ 5

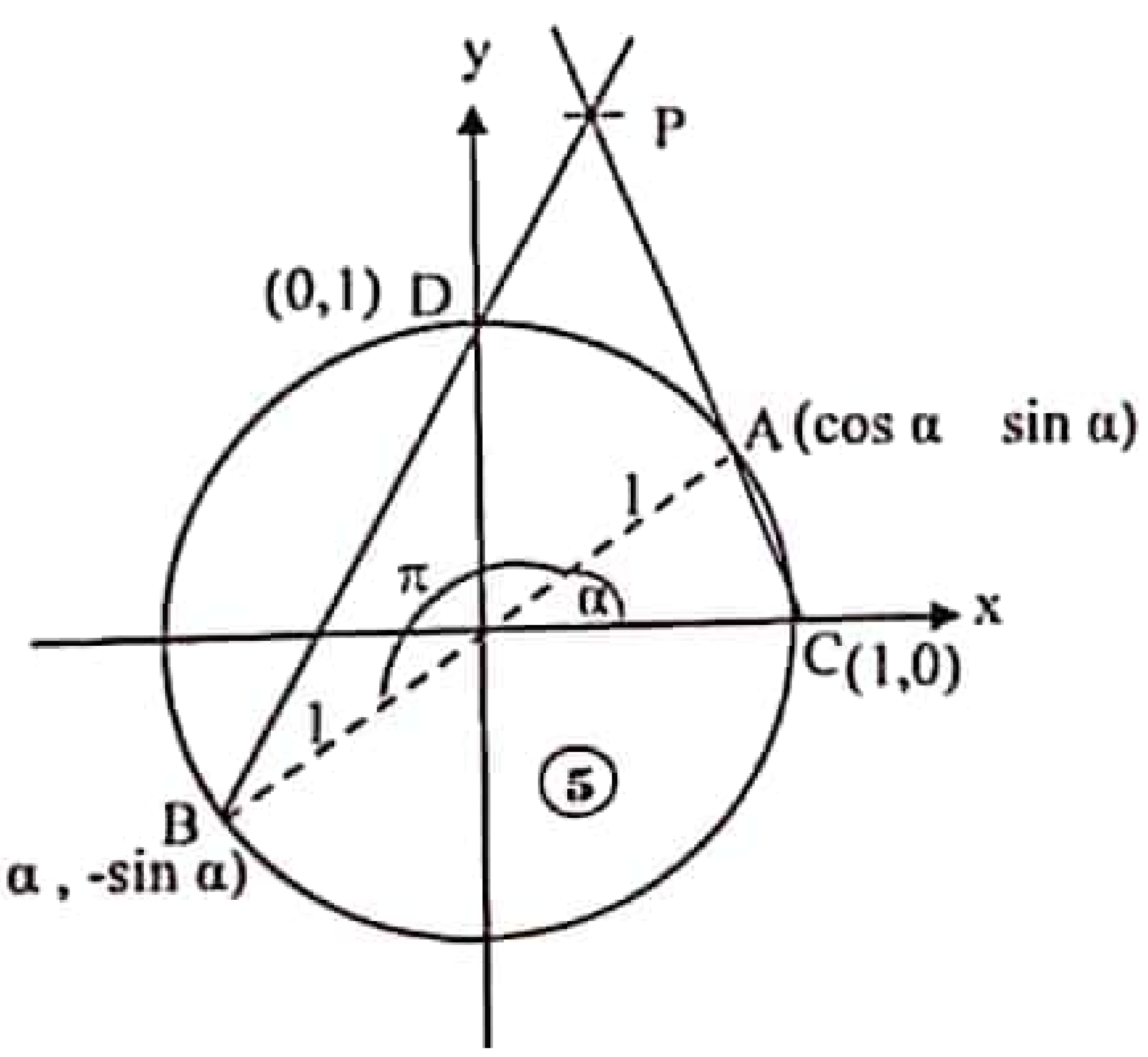
R සඳහා පිහිටීම් 2 ඇත. 5

$R \equiv \left(\frac{142}{45}, \frac{-28}{15}\right)$ 5

$R \equiv \left(\frac{-2}{45}, \frac{8}{15}\right)$

$\boxed{45}$

(b)



$B\,\{\cos(\pi + \alpha), \sin(\pi + \alpha)\}$

$B = (-\cos\alpha, -\sin\alpha)$ 5 $\boxed{10}$

A C හිසමීකරණය

$\frac{y-0}{x-1} = \frac{\sin\alpha - 0}{\cos\alpha - 1}$ 5

$$\frac{y}{x-1} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{-2\sin^2\frac{\alpha}{2}}$$ 5 5

$y\sin\frac{\alpha}{2} = (1-x)\cos\frac{\alpha}{2}$ --- (1) 5

$\boxed{20}$

BD හිසමීකරණය

$$\frac{y-1}{x-0} = \frac{1+\sin\alpha}{0+\cos\alpha}$$ 5

$$\frac{y-1}{x} = \frac{(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2})^2}{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}$$ 5 5

$$\frac{y-1}{x} = \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}}$$ 5

$(y-1)\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right) = x\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)$

$(x+y-1)\sin\frac{\alpha}{2} = (y-x-1)\cos\frac{\alpha}{2}$ -- (2) 5

$\boxed{25}$

$\frac{①}{②}$ න් $\frac{y}{x+y-1} = \frac{1-x}{y-x-1}$ 5

$y(y-x-1) = (1-x)(x+y-1)$

$x^2 + y^2 - 2x - 2y + 1 = 0$ 10

$\boxed{15}$

17. (a) $\sin(A-B) = \sin A \cos B - \cos A \sin B$ (5)

$\cos(A-B) = \cos A \cos B - \sin A \sin B$ (5)

$\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B - \sin A \sin B}$ (5)

$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$ (5)

$B \rightarrow (-B)$

$\tan[A-(-B)] = \frac{\tan A - \tan(-B)}{1 + \tan A \tan(-B)}$

$\tan[A+B] = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ (5)

$A = 2\theta \, , B = 3\theta$

$\tan[2\theta + 3\theta] = \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta}$ (5)

$\tan 2\theta + \tan 3\theta - \tan 5\theta = -\tan 2\theta \tan 3\theta \tan 5\theta$

$\tan 2\theta + \tan 3\theta - \tan 5\theta = 0$

$-\tan 2\theta . \tan 3\theta . \tan 5\theta \quad = 0$ (5)

$\tan 2\theta = 0$	$\tan 3\theta = 0$	$\tan 5\theta = 0$
$2\theta = n\pi$	$3\theta = n'\pi$	$5\theta = n''\pi$
$\theta = \frac{n\pi}{2}$	$\theta = \frac{n'\pi}{3}$	$\theta = \frac{n''\pi}{5}$ -- (5)

$n \in$ ඉරට්ටේ නිඛිල (5) $\quad n', n'' \in \mathbb{Z}$ (5)

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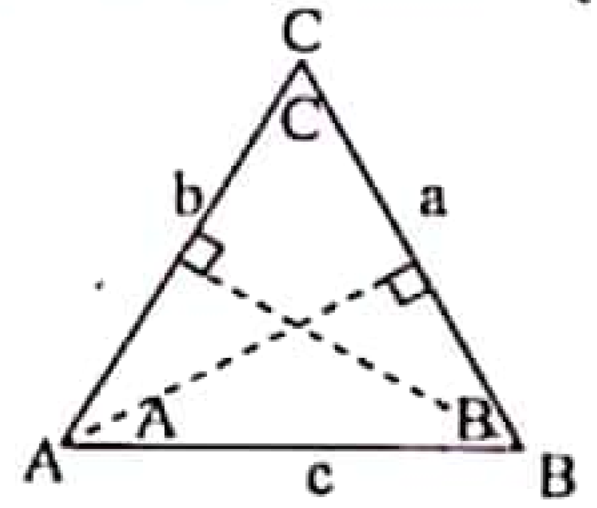
(b) සුළුකෝණි ත්‍රිකෝණ සඳහා

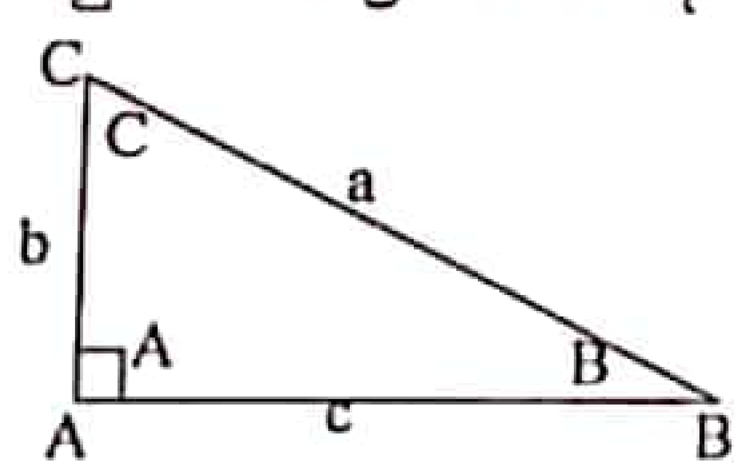


$b \sin C = c \sin B \qquad c \sin A = a \sin C$

$\frac{b}{\sin B} = \frac{c}{\sin C}$ (5) $\quad \frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

සෘජු කෝණි ත්‍රිකෝණ සඳහා



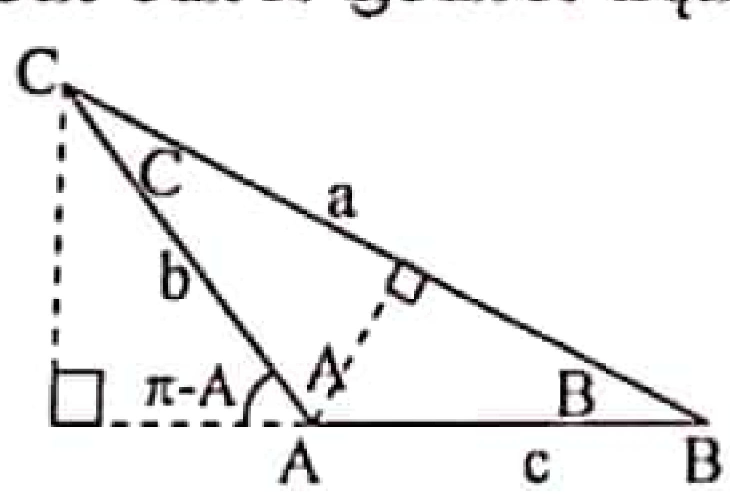
$a \sin B = b \qquad c = a \sin C$

$\frac{a}{1} = \frac{b}{\sin B} \qquad \frac{c}{\sin C} = \frac{a}{\sin \frac{\pi}{2}}$

$\frac{a}{\sin \frac{\pi}{2}} = \frac{b}{\sin B}$ (5) $\quad \frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

මහා කෝණි ත්‍රිකෝණ සඳහා



$a \sin B = b \sin(\pi - A)$ (5) $\quad b \sin C = c \sin B$

$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(රූප 3 ම සඳහා ලකුණු 5)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda$ යැයි ගනිමු. 5

$\frac{b^2+c^2-a^2}{2bc} = \frac{\sin^2 B + \sin^2 C - \sin^2 A}{2\sin B \sin C}$ 5

$= \frac{\sin^2 B + (\sin C - \sin A)(\sin C + \sin A)}{2\sin B \sin C}$

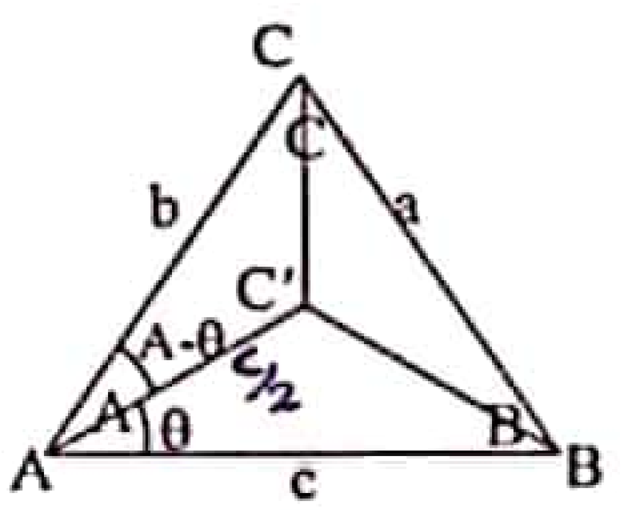
$= \frac{\sin^2 B + 2\cos\frac{C+A}{2}\sin\frac{C-A}{2} \times 2\sin\frac{C+A}{2}\cos\frac{C-A}{2}}{2\sin B \sin C}$ 5

$= \frac{\sin^2 B + \sin(C+A).\sin(C-A)}{2\sin B \sin C}$

$= \frac{\sin B[\sin B + \sin(C-A)]}{2\sin B \sin C}$ 5

$= \frac{\sin(A+C) + \sin(C-A)}{2\sin C}$

$= \frac{2\sin C\cos A}{2\sin C} = \cos A$ 5



$CAC'\Delta$ සඳහා කොස් සූත්‍රයෙන්

$(CC')^2 = b^2 + \frac{c^2}{4} - 2b\frac{c}{2}\cos(A-\theta)$ 5

$= b^2 + \frac{c^2}{4} - bc(\cos A\cos\theta - \sin A\sin\theta)$ 5

$= b^2 + \frac{c^2}{4} - bc\left(\frac{b^2+c^2-a^2}{2bc}\right)\cos\theta - 2\Delta\sin\theta$ 5

$= b^2 + \frac{c^2}{4} - \frac{1}{2}\left(b^2 + c^2 - a^2\right)\cos\theta - 2\Delta\sin\theta$

$\Delta = \frac{1}{2}bc\sin A$

$2\Delta = bc\sin A$ 5

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(c) $\alpha = \sin^{-1}(\cos x)$ $\quad\beta = \cos^{-1}(\sin x)$

$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$, $0 \le \beta \le \pi$ 5

$\alpha + \beta = \frac{\pi}{2}$

$\sin\alpha = \sin\left(\frac{\pi}{2} - \beta\right)$ 5

$\cos x = \sin x \qquad \cos x \neq 0$ 5

$\tan x = 1$ 5

$\tan x = \tan\frac{\pi}{4}$

$x = n\pi + \frac{\pi}{4}$, $n \in z$ 5

$x = \frac{-3\pi}{4} \quad \sin\alpha = \cos\left(\frac{-3\pi}{4}\right) = -\cos\frac{\pi}{4}$

$\sin\alpha = \frac{-1}{\sqrt{2}} = \sin\left(-\frac{\pi}{4}\right) \quad \alpha = -\frac{\pi}{4}$ 5 $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

$\cos\beta = \cos\left(\frac{-3\pi}{4}\right) = -\sin\frac{\pi}{4}$

$\cos\beta = \frac{-1}{\sqrt{2}} = \cos\left(\pi - \frac{\pi}{4}\right)$

$\beta = \frac{3\pi}{4}$ 5 $\qquad 0 \le \beta \le \pi$

(නැතහොත් $\alpha + \beta = \frac{\pi}{2}$ විට $\beta = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$)

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